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Enthalpy Damping for High Mach Number Euler Solutions

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Abstract

AN improvement on the enthalpy damping procedure currently in use in solving supersonic flowfields is described. A correction based on entropy values is shown to produce a very efficient scheme for simulation of high Mach number three-dimensional flows. Substantial improvements in convergence rates have been achieved by incorporating this enthalpy damping scheme in a finite-volume Runge-Kutta method for solving the Euler equations. Results obtained for blended wing-body geometries at high Mach numbers are presented.

Contents

High Mach number Euler solutions have gained importance in recent years because of the opening up of a number of application areas including the National Aerospace Plane and the high-speed civil transport concepts. A finite-volume explicit algorithm[1] has enjoyed wide applicability in such problem areas. The algorithm, when originally developed for predominantly transonic flow solutions, incorporated an enthalpy damping technique for accelerating the convergence of the time-stepped solution to a steady state. Tnis enthalpy damping technique, in its original formulation, contained ad hoc terms applied to the energy equation and has been generally found to be untenable in supersonic flow computations.

A detailed analysis presented in Ref. 2 established that this enthalpy damping procedure is effective only in locally subsonic flows. In order to render the technique applicable in supersonic flow computations, it was found in Ref. 2 that special attention needed to be given to the energy equation. Instead of ignoring the energy equation and applying the ad hoc procedure, the following enthalpy equation was appended to the system of governing equations:

$$\frac{\partial h}{\partial t} + L_{4h} = -\beta(h - h_0) \tag{1}$$

Thus, instead of ignoring the entropy, an attempt is made to drive the total enthalpy to its steady-state value. Eliminating h, Eq. (1) can be rewitten as

$$\frac{\partial E}{\partial t} + L_{4E} = -\left(\alpha E + \frac{\beta \rho}{\gamma}\right)(h - h_0) \tag{2}$$

On the basis of this modified technique, it was concluded in Ref. 2 that the enthalpy damping technique was effective because of the energy equation and not the continuity equation. This improved technique could be used equally well in supersonic regions. The coefficient α must be zero in supersonic regions, but β can be used for both subsonic and supersonic flow and its value does not depend on the Mach number. This technique has been used successfully in supersonic flow computations and results have been presented in Ref. 2.

The enthalpy damping technique just described could not be used efficiently in flow computations at Mach numbers exceeding 3.0. The finite-volume Runge-Kutta code became increasingly unstable as the Mach number was increased and could be run only at very small Courant-Friedrichs-Lewy (CFL) numbers. The cause of this instability was traced to rather large amounts of entropy generated in the flowfield in the transient stages of the iteration process at very high Mach numbers. The Gibbs equation relating the entropy to other flow properties may be written as[3]

$$\mathrm{d}h = T\,\mathrm{d}s + v\,\mathrm{d}P \tag{3}$$

Where h, s, and v are the specific enthalpy, specific entropy and specific volume, respectively. P denotes pressure and T is the absolute temperature. In flowfields containing strong shocks, the generated entropy $\mathrm{d}s$ is rather large and, consequently, the contribution $T\mathrm{d}s$ to $\mathrm{d}h$ is also large. At intermediate stages of the iteration when strong shocks are being generated, this contribution to $\mathrm{d}h$ due to changes in entropy is an integral property of the evolving flowfield, and quick damping of the total enthalpy difference introduces a physical inconsistency in the transient flow. Therefore, the flowfield may be stabilized by refraining from damping the $T\mathrm{d}s$ term at early stages of the iteration process. In the steady state, the difference between local and freestream enthalpy values must vanish and, therefore, the whole of $\mathrm{d}h$ including the $T\mathrm{d}s$ part has to be damped at later stages of the iteration process. To this end, the term $h - h_0$ in Eq. (2) is replaced by $h - h_0 - \delta T\,\mathrm{d}s$ such that Eq. (2) becomes

$$\frac{\partial E}{\partial t} + L_{4E} = -\left(\alpha E + \frac{\beta \rho}{\gamma}\right)(h - h_0 - \delta T\,\mathrm{d}s) \tag{4}$$

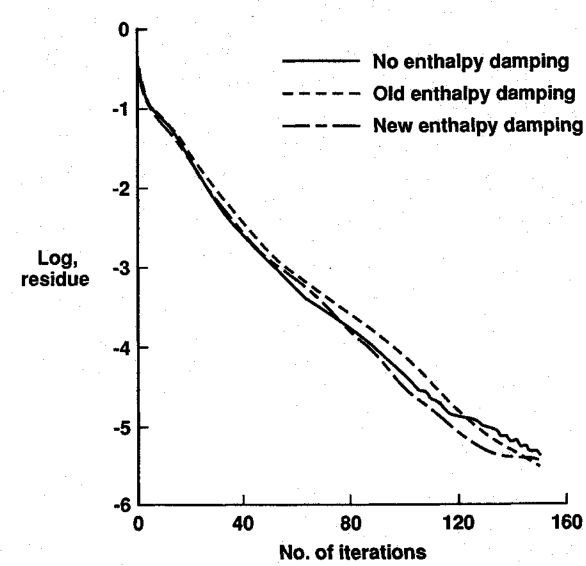

Fig. 1 Comparison of convergence histories, blended wing body: $M = 3.0$, $\alpha = 2.0$ deg, CFL = 4.0.

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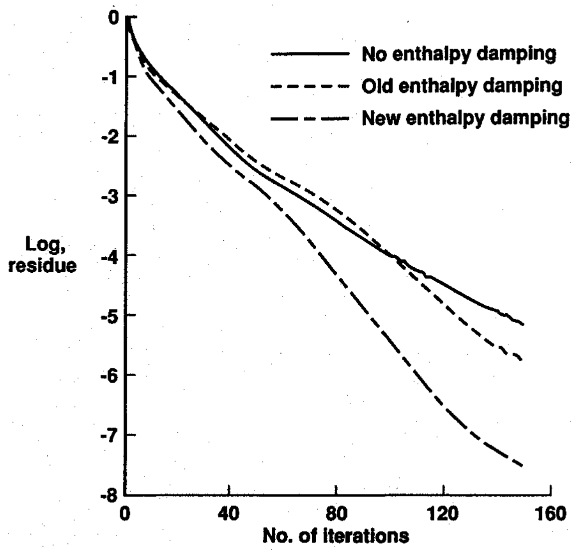

Fig. 2 Comparison of convergence histories, 6-deg cone: $M = 6.0$, $\alpha = 2.0$ deg, CFL = 4.0.

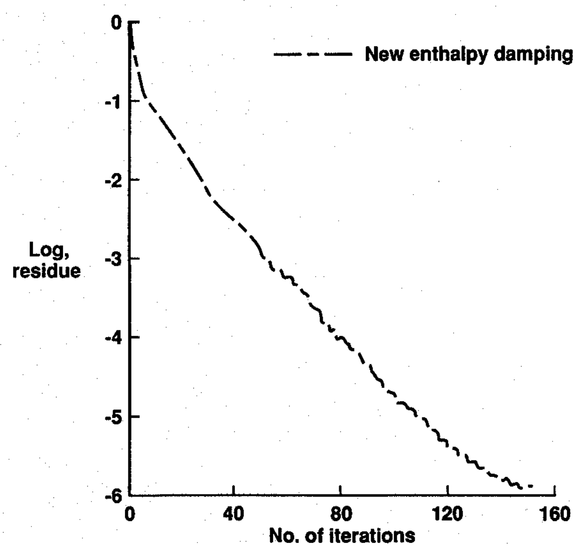

Fig. 3 Comparison of convergence histories, blended wing body: $M = 6.0$, $\alpha = 2.0$ deg, CFL = 4.0.

where δ is a coefficient that varies from 1 at the beginning of the iteration process to 0 at the end of the iteration. The parameter δ was prescribed as a simple linear function of the number of iterations in order to decay its value from 1 to 0 over the first 100 iterations. The flow computations are normalized with $P = 1$ and $\rho = 1$ at infinity; therefore, the quantity $S = P/\rho^\gamma - 1$ may be used as a measure of entropy generation to be used in computing $T\,ds$ in Eq. (4).

All of the test cases, for which results are presented later, involved purely supersonic flow. The coefficient α was therefore set to zero for all of these cases. The value of β varied from 0.1 at $M = 3.0$ to 0.5 at $M = 8.0$. The present enthalpy damping technique was designed specifically for high Mach number flows containing very strong shocks, and, consequently, the benefits derived from its application are not expected to be substantial for low Mach number flows ($M \leq 3.0$). The first test case at $M = 3.0$ shows this to be true. The convergence plots presented in Fig. 1 are for calculations for a three-dimensional blended wing-body geometry at a Mach number of 3.0 and an angle of attack of 2.0 deg. Convergence histories are plotted for three cases: 1) with no enthalpy damping, 2) with the old enthalpy damping technique for supersonic flows, and 3) with the present formulation of enthalpy damping. As expected, the convergence levels attained after 150 iterations are identical for all three cases.

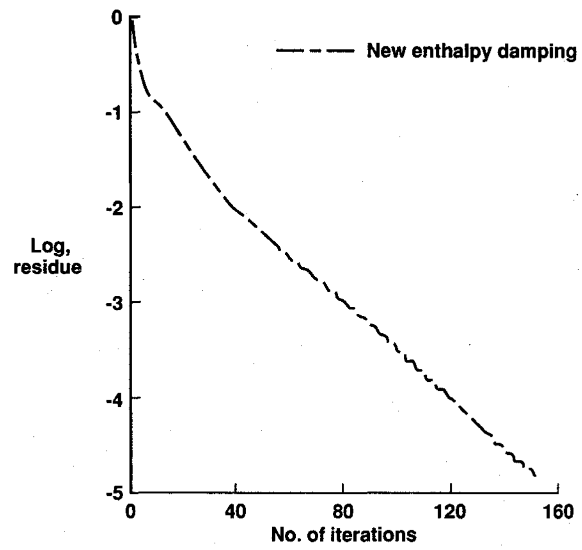

Fig. 4 Comparison of convergence histories, blended wing body: $M = 8.0$, $\alpha = 2.0$ deg, CFL = 3.0.

An increase in the Mach number is expected to result in significant gain in the rate of convergence; this is evidenced in the next test case. Flow about a sharp-nosed cone with half-angle = 6.0 deg, at $M = 6.0$ and $\alpha = 2.0$ deg, was computed; the convergence histories resulting from the three different enthalpy damping cases are presented in Fig. 2. It may be clearly seen that the use of the new damping technique results in two additional orders of reduction in the residual over the convergence resulting from the older version of damping. The increase in reduction of the residual over no damping is close to 2.5 orders of magnitude. This clearly validates the usefulness of the new enthalpy damping technique.

A blended wing-body configuration at $M = 6.0$ was the next test case. The old damping technique was found to be destabilizing for this case at CFL = 4.0 and, therefore, could not be used. The new technique, however, worked very efficiently and resulted in almost 6 orders of reduction of the computed residual in only 150 iterations. The convergence history for this case is presented in Fig. 3.

The Mach number was increased to 8.0 for the next test case. For a CFL a value of 3.0, the old damping is again seen to be destabilizing and only the new damping could be used resulting in about 5 orders of reduction of the residual in 150 iterations, as shown in Fig. 4.

All of the blended wing-body cases presented here were computed on a grid containing 65 sections along the longitudinal axis, 33 points in the circumferential direction in each sectional grid, and 49 points in the radial direction. Some studies were performed to investigate the effects of refining the grid in the radial and the circumferential directions. Relative gains in the convergence rates were noted to be larger for the more refined grids. This is attributable to the finer resolution of the shocks and the consequent higher accuracy of prediction of the associated entropy rises. Numerical experimentation with a wide variety of configurations at high Mach numbers has shown that the new enthalpy damping technique is a useful tool for high Mach number computations at high CFL numbers. The present technique provides stable and fast convergence at high CFL numbers in cases involving complex geometries where the old damping technique may be inoperable.

References

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